

L^p inequalities

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190523 BBS:Itai-190523+: L^p inequalities: $\|u\|_q \leq C_{p,d} \|\nabla u\|_p$, (in \mathbb{R}^d)
where $\frac{1}{q} = \frac{1}{p} - \frac{1}{d}$; $\|u\|_p \leq \|u\|_{p_0}^{1-\theta} \|u\|_{p_1}^\theta$.

$L^p(\mathbb{R})$: That which barely rejects $x^{-1/p}$
near 0, if $p' < p$, then $x^{-1/p} \in L^{p'}$

$L^p(\mathbb{R}^d)$: That which barely rejects $x^{-d/p}$

$$\nabla(x^{-d/q}) \sim x^{-\frac{d}{q}-1} = x^{-d/p}$$

$$\Rightarrow \frac{d}{q} + 1 = \frac{d}{p} \Rightarrow \frac{1}{q} + \frac{1}{d} = \frac{1}{p}$$